Determination of the wavelengths of the Sodium doublet lines and the measurement of the thickness of a thin transparent film using a Michelson interferometer

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ABSTRACT

This experiment aims to discern the wavelengths of the two sodium doublet lines[1] by the use of a Michelson-interferometer, and then to discern the thickness of a sample of Mylar film utilising the same equipment.

The following values for the wavelength of the sodium doublet lines were calculated:

$$\lambda_1 = (589.01 \pm 0.01) \times 10^{-9} \text{ m}$$
$$\lambda_2 = (589.59 \pm 0.01) \times 10^{-9} \text{ m}$$

These values are remarkably close to the accepted values for the two sodium D-lines[1] of:

$$\lambda_{A1} = 588.99 \times 10^{-9} \text{ m}$$
$$\lambda_{A2} = 589.59 \times 10^{-9} \text{ m}$$

The thickness of the Mylar film was found to be:

$$t = (1.66 \pm 0.06) \times 10^{-5} \text{ m}$$

Which is to the same order as the manufacturer specified thickness of the Mylar of:

$$(1.3 \pm 0.1) \times 10^{-5} \text{ m}$$

This experiment could be considered a success. There are, however, a number of improvements which could be made to improve the rather large level of human error involved in the procedures performed.

[1] Also known as the Fraunhofer D-lines.

1 Operation of the Michelson Interferometer

A schematic view of a Michelson interferometer is depicted in Figure 1.1. Light from the source enters the interferometer and encounters a beam splitter. A portion of the light continues on to reflect from mirror M_2, this reflected from the beam splitter into the detector. The other portion of light is reflected by the beam splitter towards the movable mirror M_1, this mirror can be adjusted to increase or decrease the optical path difference between the paths taken by the two fractions of the initial light. Having reflected from mirror M_1 this light passes back through the beam splitter to the observer. A more detailed description of the apparatus and the theory behind it can be found in Michelson and Morley's original paper from it's use in attempting to discover the "luminiferous ether"[1], the script for this experiment[2] or in various text books.

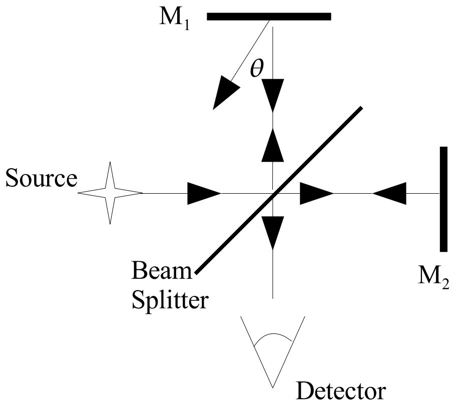

Figure 1.1: Schematic of a Michelson interferometer. The light path is shown by the arrowed lines.

[1] [2]
[2] [3]

2 Calibrating the apparatus

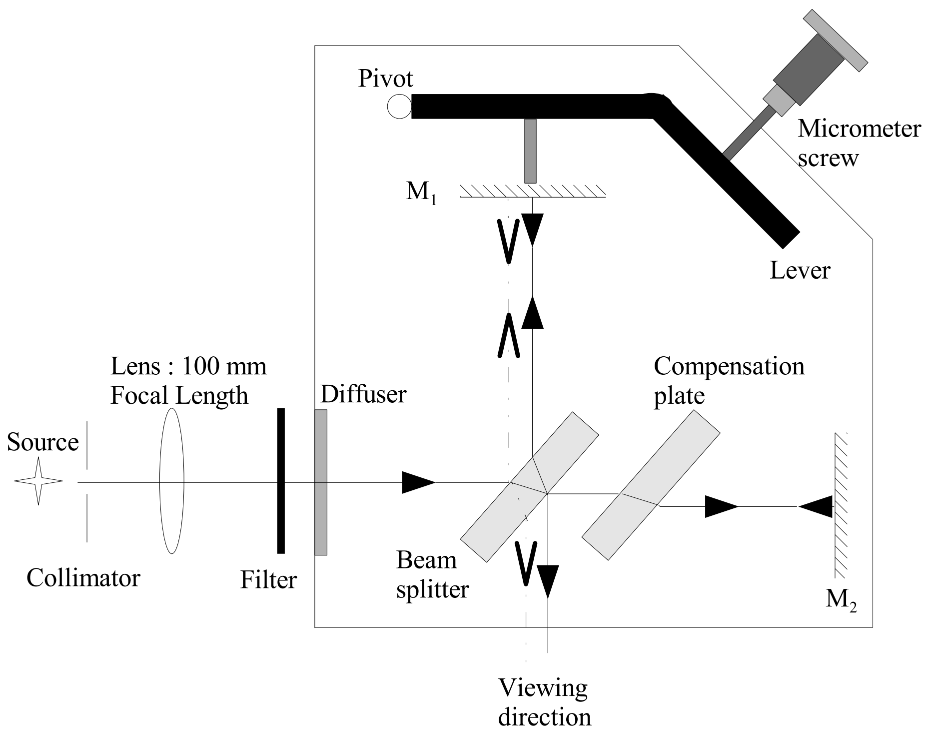

Figure 2.1: The experimental apparatus. The light path is shown as a solid line, with the dashed line indicating a small amount of reflected light.

Figure 2.1 shows the apparatus used in the experiment. Notice the addition of the compensation plate, this is necessary to account for the phase change and extra optical path added by the beam splitter due to it not being infinitesimally thin[3]. The filter is not necessary in all parts of the experiment, only when a certain wavelength of light needs to be examined. The most obvious addition in the experimental set up, however, is the use of a lever to adjust the mirror, this necessitates the finding of a correction factor between the distance moved on the micrometer screw ggaugeand the corresponding change in the optical path length caused by the mirror movement.

2.1 Adjustment for circular fringes

Before measurements can be taken, it is necessary to perform a calibration of the apparatus. Firstly, an adjustment for circular fringes[3] is made, this brings a set of interference fringes into the field of view. A diagrammatic view of the interference fringes obtained is shown in Figure 2.2.

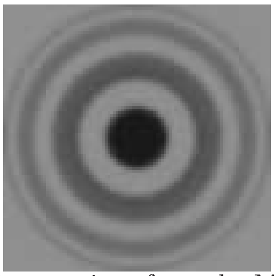

Figure 2.2: Interference rings from the Michelson interferometer.

2.2 Calibration of mirror movement

For this part of the experiment a mercury lamp is used as the source for the Michelson interferometer and a filter isolating light of wavelength 546.07 nm is used. The procedure used is detailed in the experiment script[1].

As was mentioned before, the use of a lever arm to move mirror M_1 necessitates the determination of a correction factor to ascertain the actual actual change in the optical path length caused by the movement of the micrometer screw gauge. This was determined by the method suggested in the script[3] with the one exception of allowing 100 instead of 200 fringes to pass due to repeated problems with backlash on the micrometer screw gauge, more measurements were taken to ensure the accuracy of the experiment did not suffer. Since every fringe passing indicates a change in the optical path length, Δd, of half a wavelength, $\frac{\lambda}{2}$, and a filter selecting only light with a wavelength of 546.07 nm was used, in this case the correction factor, f, can be found using:

$$f = \frac{\Delta d}{\Delta L} = \frac{2.73035 \times 10^{-9}}{\Delta L} \tag{2.1}$$

Where ΔL is the distance moved on the micrometer.

Table 2.1 contains the calibration data obtained, with the errors calculated using the general error propagation equation[4]:

$$(\Delta f)^2 = \left(\frac{\delta f}{\delta x_1}\right)^2 (\Delta x_1) + \left(\frac{\delta f}{\delta x_2}\right)^2 (\Delta x_2) + ... + \left(\frac{\delta f}{\delta x_n}\right)^2 (\Delta x_n) \tag{2.2}$$

[1] [3]

Starting micrometer reading L_i / $\times 10^{-3}$ m	Error on starting micrometer reading ΔL_i / $\times 10^{-3}$ m	Final micrometer reading L_f / $\times 10^{-3}$ m	Error on final micrometer reading ΔL_f / $\times 10^{-3}$ m	Difference between readings ΔL / $\times 10^{-3}$ m	Error on difference between readings $\Delta\Delta L$ / $\times 10^{-3}$ m
8.00	0.01	7.87	0.01	0.13	0.01
8.00	0.01	8.13	0.01	0.13	0.01
7.00	0.01	6.87	0.01	0.13	0.01
7.00	0.01	6.88	0.01	0.12	0.01
6.50	0.01	6.36	0.01	0.14	0.01
6.50	0.01	6.36	0.01	0.14	0.01

Table 2.1: Calibration data for finding the correction factor.

Using equation 2.1, and finding the error using equation 2.2, the correction factor can be found to be:

$$f = (2.08 \pm 0.07) \times 10^{-1} \text{ (no units)}$$

3 Determination of the wavelength of the Sodium doublet lines and the thickness of a Mylar film

3.1 Separation of the sodium doublet lines

3.1.1 Measurement of points of zero optical path difference

For this part of the experiment the filter is removed from the apparatus and a sodium lamp is used as the light source for the Michelson interferometer. The procedure to be used is detailed in the script for the experiment[1].

It can be found that for light of two very similar wavelengths (as we have here) the difference between the wavelengths is[3]:

$$\begin{aligned} \Delta\lambda &= \frac{{\lambda_m}^2}{2\Delta d} \\ &= \frac{{\lambda_m}^2}{2f\Delta L} \end{aligned} \tag{3.1}$$

Where:

$\Delta\lambda$ is the difference in wavelength between the doublet lines.

λ_m is the mean wavelength of the doublet (in this case[3] 589.3 nm)

The micrometer movements, ΔL, will be so as to move the mirror between two points of zero optical path difference. This is signified by the disappearance of the interference fringes and the appearance of a uniform colour, as seen in Figure 3.1.

By measuring the distance between successive states of zero optical path difference, a value for the wavelength difference between the two sodium doublet lines can be found and, since the mean wavelength is known, the wavelengths of them can be deduced.

3.1.2 Results

Table 3.1 contains measurements of the micrometer distance between points of zero path difference, with the errors again determined using equation 2.2.

[1] [3]

Figure 3.1: Diagrammatic view of the Michelson interferometer at a state of zero optical path difference.

Starting micrometer reading L_i / $\times 10^{-3}$ m	Error on starting micrometer reading ΔL_i / $\times 10^{-3}$ m	Final micrometer reading L_f / $\times 10^{-3}$ m	Error on final micrometer reading ΔL_f / $\times 10^{-3}$ m	Difference between readings ΔL / $\times 10^{-3}$ m	Error on difference between readings $\Delta\Delta L$ / $\times 10^{-3}$ m
2.41	0.01	3.47	0.01	1.06	0.01
3.47	0.01	4.94	0.01	1.47	0.01
4.94	0.01	6.48	0.01	1.54	0.01
6.48	0.01	8.02	0.01	1.54	0.01
8.02	0.01	9.48	0.01	1.46	0.01
9.48	0.01	10.92	0.01	1.44	0.01

Table 3.1: Sodium doublet lines : Micrometer readings between points of zero path difference.

Averaging ΔL and using equation 3.1 the difference in wavelength of the sodium doublet lines (with an error found using equation 2.2) is found to be:

$$\Delta\lambda = (5.8 \pm 0.2) \times 10^{-10} \text{ m}$$

Therefore, since the average wavelength of the sodium doublet lines is known to be 589.3 nm, the wavelengths of the two sodium doublet lines (with an error found using equation 2.2) must be:

$$\lambda_1 = (589.01 \pm 0.01) \times 10^{-9} \text{ m}$$
$$\lambda_2 = (589.59 \pm 0.01) \times 10^{-9} \text{ m}$$

3.2 Determination of the thickness of a thin transparent film (Mylar)

3.2.1 Adjustment for white light fringes

In this part of the experiment no filter is used and a white light is used as the source for the Michelson interferometer. The procedure to be used is detailed in the script for the experiment[2].

A set of straight fringes is observed with a central dark fringe indicating the point of zero path difference, as is seen in figure 3.2.

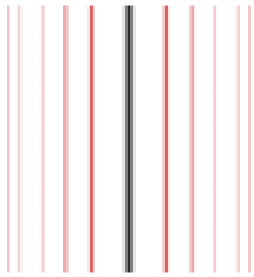

Figure 3.2: Diagrammatic view of the straight white light fringes with a central black fringe at the centre showing the point of zero path difference. *Note: The fringes shown in red are not necessarily red when viewed they are just shown as red to distinguish between coloured and black fringes here.*

3.2.2 Calculation of the optical path introduced by the film

Again, a white light source and no filter are used in the part of the experiment. The procedure to be used is detailed in the script for the experiment[3].

When the piece of Mylar is inserted into an arm of the interferometer it will cause an increase in the optical path. By finding the position of zero path difference without the Mylar film inserted (as in section 3.2.1) and then measuring the change needed to once again obtain a state of zero path difference with the Mylar inserted, it is possible to determine the optical path difference of the Mylar film.

Once the optical path difference of the film is known, and knowing that the refractive index of Mylar[3] is 1.64, the thickness of the Mylar film can be found using the

[2] [3]
[3] [3]

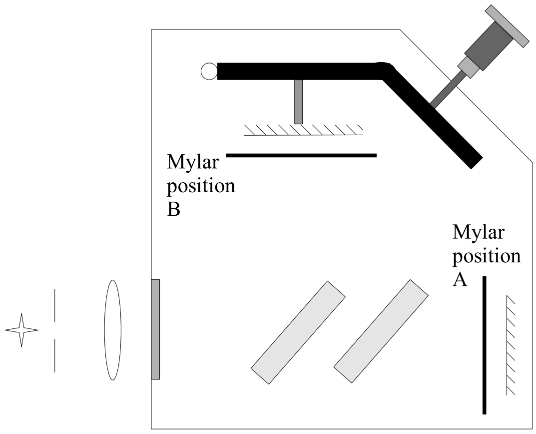

Figure 3.3: Showing positions A and B of the Mylar film in the arms of the Michelson interferometer.

equation:

$$\begin{aligned} \Delta p &= 2t_{\text{Mylar}}\left(n_{\text{Mylar}} - n_{\text{air}}\right) \\ \Delta d &= t_{\text{Mylar}}\left(n_{\text{Mylar}} - n_{\text{air}}\right) \\ t_{\text{Mylar}} &= \frac{\Delta d}{\left(n_{\text{Mylar}} - n_{\text{air}}\right)} \end{aligned} \tag{3.2}$$

Where:

Δp is the change in optical path difference.
n_{Mylar} is the refractive index of Mylar[3] (1.64)
n_{air} is the refractive index of air. (1.00)
t_{Mylar} is the thickness of the Mylar film.

The Mylar will be placed in two different arms of the Michelson interferometer, A and B, as depicted in figure 3.3.

3.2.3 Results

Tables 3.2 and 3.3 show the changes needed to the micrometer screw gauge to get back to a state of zero path length once the Mylar had been inserted into positions A and B respectively (with errors calculated using equation 2.2).

By averaging the changes on the micrometer screw gauge, finding the change in path length caused by these changes on the micrometer screw gauge, averaging again, and finding errors using equation 2.2 it is found that:

$$\Delta d = (1.06 \pm 0.06) \times 10^{-5} \text{ m}$$

Starting micrometer reading L_i / $\times 10^{-3}$ m	Error on starting micrometer reading ΔL_i / $\times 10^{-3}$ m	Final micrometer reading L_f / $\times 10^{-3}$ m	Error on final micrometer reading ΔL_f / $\times 10^{-3}$ m	Difference between readings ΔL / $\times 10^{-3}$ m	Error on difference between readings $\Delta\Delta L$ / $\times 10^{-3}$ m
7.24	0.01	7.29	0.01	0.05	0.01
7.24	0.01	7.29	0.01	0.05	0.01
7.24	0.01	7.29	0.01	0.05	0.01
7.24	0.01	7.29	0.01	0.05	0.01
7.24	0.01	7.29	0.01	0.05	0.01
7.24	0.01	7.29	0.01	0.05	0.01

Table 3.2: Movement of micrometer screw gauge required to obtain zero path length again once the Mylar has been inserted into position A.

Now, using equation 3.2, and again finding errors using equation 2.2, the thickness of the Mylar can be found to be:

$$t = (1.66 \pm 0.06) \times 10^{-5} \text{ m}$$

Starting micrometer reading L_i / $\times 10^{-3}$ m	Error on starting micrometer reading ΔL_i / $\times 10^{-3}$ m	Final micrometer reading L_f / $\times 10^{-3}$ m	Error on final micrometer reading ΔL_f / $\times 10^{-3}$ m	Difference between readings ΔL / $\times 10^{-3}$ m	Error on difference between readings $\Delta\Delta L$ / $\times 10^{-3}$ m
7.24	0.01	7.19	0.01	0.05	0.01
7.24	0.01	7.18	0.01	0.06	0.01
7.24	0.01	7.19	0.01	0.05	0.01
7.24	0.01	7.19	0.01	0.05	0.01
7.24	0.01	7.19	0.01	0.05	0.01
7.24	0.01	7.19	0.01	0.05	0.01

Table 3.3: Movement of micrometer screw gauge required to obtain zero path length again once the Mylar has been inserted into position B.

4 Conclusion

4.1 Aims and Objectives

The aims of this experiment were to use a Michelson interferometer to determine the wavelength of the two sodium doublet lines and to determine the thickness of thin film of Mylar.

4.2 Summary of results

4.2.1 The wavelength of the sodium doublet lines

The wavelengths of the two sodium doublet lines were found to be:

$$\lambda_1 = (589.01 \pm 0.01) \times 10^{-9} \text{ m}$$
$$\lambda_2 = (589.59 \pm 0.01) \times 10^{-9} \text{ m}$$

With the accepted values being[1]:

$$\lambda_{A1} = 588.99 \times 10^{-9} \text{ m}$$
$$\lambda_{A2} = 589.59 \times 10^{-9} \text{ m}$$

The results gained here would look to be very close to these, with the exception of the error bar of λ_1 not encompassing the accepted value, λ_{A1}.

4.2.2 The thickness of a thin film (Mylar)

The thickness of the Mylar film was found to be:

$$t = (1.66 \pm 0.06) \times 10^{-5} \text{ m}$$

Which, although being close (i.e. to the same order as), deviates slightly from the manufacturers specified thickness of:

$$(1.3 \pm 0.1) \times 10^{-5} \text{ m}$$

4.3 Problems and errors introduced during the experiment and how to correct them

The main source of error in this experiment would seem to be a human one, it is exceptionally difficult to be as accurate as would be preferred when attempting to make as subtle movements on the micrometer screw gauge as were needed.

4.3.1 Calculating the correction factor, f

Problems and errors

The main problem here was backlash from the micrometer screw gauge, a problem which resulted in the changing of the original plan for the measurement procedure[1]. Another problem encountered was the possibility of over or under counting the amount of fringes that had passed due to a small movement of the micrometer screw gauge causing a large number of fringes to pass. Finally it was highly possible to knock the micrometer screw gauge when releasing ones grip on it, causing another error.

Unfortunately it is very hard, if not impossible, to put a number on these human errors and as such whilst they are definitely there they are hard to take into account when determining a final answer.

Correcting these errors

The best method, it would seem, to removing these errors would be to have a computer connected to a camera and motor assembly take the readings. Using a motor would prevent backlash and other problems with the micrometer screw gauge, while having a computer count the fringes would reduce counting errors. This would help remove the element of human element present and make any errors occurring easier to place a numerical value on.

4.3.2 Finding the separation of the sodium doublet lines

Problems and errors

The problem with backlash was again a factor here, as was knocking the micrometer screw gauge when releasing ones grip on it. Another problem was that it was hard to discern the exact point of zero path difference, when the fringes were totally invisible.

Again these are mainly human errors and are very hard, if not impossible, to put a numerical value on.

Correcting these errors

A computer with a motor and camera assembly would again do much to solve these problems, both for the reasons specified above, and for the reason that a computer program could determine the point of zero path difference far more effectively than human sight.

[1] [3]

4.3.3 Determination of the thickness of the Mylar film

Problems and errors

When adjusting for white light fringes it was quite difficult to judge when the lines were perfectly straight which could give an error in the results. Once again the problems with backlash and knocking the micrometer screw gauge are an issue. Another issue was with the Mylar being creased in it's holder, thus giving the fringes a distorted look with an effect similar to oil on water (see figure 4.1 for an approximate diagram of this).

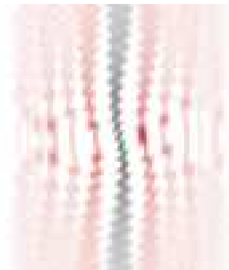

Figure 4.1: Diagrammatic representation of distortion to fringes caused by creasing of the Mylar film. *Note: Once again red fringes are only to highlight which fringes were coloured.*

The Mylar was changed for a less creased piece and this improved, the image was, however, still slightly distorted, making it difficult to judge the point of zero path difference. Stretching of the Mylar in trying to straighted it could also have affected its thickness.

Correcting these errors

Once again, a computerised system would remove a lot of error in determining the point of zero path difference. Modifications should also be made to the way the Mylar is inserted, a system of clamps at each edge of the film would possibly work better, although care would need to be taken to ensure that the Mylar film was not stretched.

Bibliography

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