

# Investigating the Zeeman Effect in Mercury

Luke Pomfrey

November 7, 2007

## Abstract

The Zeeman Effect causes the energy levels of electrons of some materials to split into a number of different levels when a sample of the material is placed in a magnetic field. [1, 2, 3] This splitting of energy levels results in a splitting of the spectral lines emitted by the sample. In this experiment the splitting of spectral lines in Mercury, by the action of a magnetic field, was investigated. Both the energy change between each line and the polarisation of those lines was found and compared to theory.

The results of the experiment show that the polarisation of the lines is indeed governed by the following conditions [4, 5]:

- For  $\Delta m_j = 0$  (where  $m_j$  is the magnetic quantum number, related to the component of angular momentum in the  $z$ -direction) the photons are linearly polarised along the direction of  $\mathbf{B}$ , these are termed  $\pi$  lines.
- For  $\Delta m_j = \pm 1$  the photons are circularly polarised along the direction of  $\mathbf{B}$ , these are termed  $\sigma$  lines.

The change in energy from the central line to the line in question should be given by the following.[1, 2, 3, 4, 5]

$$\Delta E = \mu_B B \Delta (g_j m_j)$$

Where,  $\mu_B$  is the Bohr magneton,  $B$  the magnetic field strength, and  $g_j$  the  $g$ -factor. The results gained for these energy shifts are all to the same order of magnitude as the predicted values but all fall outside the error ranges on them. This would appear to have been caused by a systematic error, this will be discussed later.

# Chapter 1

## Introduction

The Zeeman effect causes the energy levels of electrons of some materials to split into a number of different levels when a sample of the material is placed in a magnetic field. [1, 2, 3, *et al.*] This splitting of energy levels results in a splitting of the spectral lines emitted by the sample.

Upon the immersion of the sample in a magnetic field each energy level splits into  $(2j + 1)$  possible energy states. [1] These states are symmetrically aligned around the original energy state.

The wavelength of light emitted by the sample is governed by these states, as the light emitted is caused by the transition of electrons between them. The frequency and wavelength of the emitted photons is given by the following equation. [5]

$$E = h\nu = \frac{hc}{\lambda} \quad (1.1)$$

Where  $E$  is the energy difference between the states,  $\nu$  the frequency of the emitted photon,  $\lambda$  the wavelength of the emitted photon,  $h$  Planck's constant, and  $c$  the speed of light.

The possible transitions are governed by the selection rules given in table 1.1. [5]

---

$\Delta s = 0$	
$\Delta l = 0, \pm 1$	
$\Delta j = 0, \pm 1$	(but not $j = 0 \rightarrow j = 0$ )
$\Delta m_j = 0, \pm 1$	(but not $m_j = 0 \rightarrow m_j = 0$ if $\Delta j = 0$ )

---

Table 1.1: Selection rules for a multi-electron atom.

It can be shown ([1, 2, 3, 4, 5]) that it is possible to calculate the predicted change in energy between the central energy level and the subsequent split levels by using the formula:

$$\Delta E = \mu_B B \Delta (g_j m_j) \quad (1.2)$$

Where,  $\mu_B$  is the Bohr magneton,  $B$  the magnetic field strength, and  $g_j$  the  $g$ -factor given by equation 1.3

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad (1.3)$$

It can be shown that ([4, 5]) that the emitted photons should be polarised. These polarisations are subject to the conditions given in table 1.2.

<b>State change</b>	<b>Polarisation direction</b>	<b>Term</b>
$\Delta m_j = 0$	Photons are linearly polarised along the direction of $\mathbf{B}$ .	$\pi$ lines.
$\Delta m_j = \pm 1$	Photons are circularly polarised along the direction of $\mathbf{B}$ .	$\sigma$ lines.

Table 1.2: Polarisation conditions for emitted photons.

## Chapter 2

# Method

The equipment shown in figure 2.1 was setup and aligned according to the method described in the script [6].

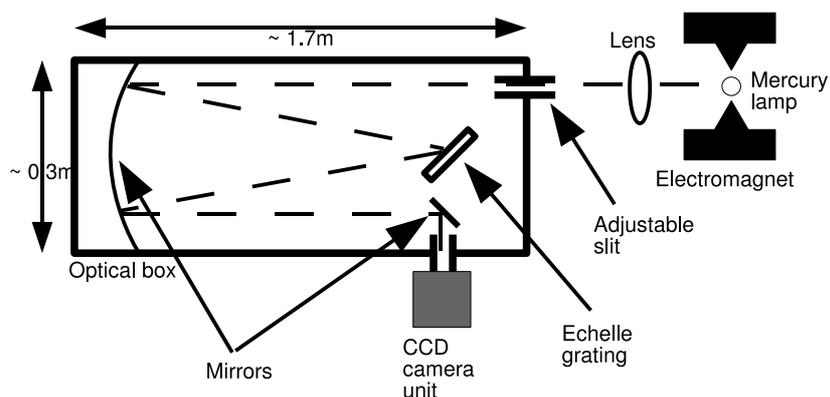


Figure 2.1: Experimental setup. (Dashed line shows light path.)

Four different coloured spectral lines were analysed in the experiment; violet, green, blue and yellow. Three (3) images were taken of each.

Images taken with the CCD camera were analysed using the C4 and MIRA software packages as described in the script [6] to obtain the values for the intensity of each pixel across a slice of the image. This data could then be input into *SigmaPlot* to obtain a graph and the distance in pixels between each line (as a peak in the graph). The distance between the lines was averaged over the 3 images of each colour spectral line.

Once the distance between each line was known it was then possible to calculate the wavelength difference between each line using the following equation. [6]

$$d\lambda \text{ (m)} = \frac{\lambda}{5.395 \times 10^5} dx \text{ (m)} \quad (2.1)$$

Where  $d\lambda$  is the wavelength difference between each line,  $dx$  the distance between the lines in pixels, and  $\lambda$  the wavelength of the central line.

From the results gained it was then possible to calculate the wavelength difference from the central line to each other line and then the energy change between the central line to each other line using. [5]

$$\Delta E = \frac{hc}{\lambda^2} \Delta\lambda \quad (2.2)$$

Where  $\Delta E$  is the energy change between the central line and the line being measured,  $h$  is Planck's constant,  $c$  is the speed of light,  $\lambda$  is the wavelength of the central line and  $\Delta\lambda$  is the wavelength difference between the central line and the line being measured.

Polarisation of each line could be determined by placing a polarising film between the mercury lamp and the lens, in one of two orientations and taking an image with the camera.

Once the polarisation and energy differences were known they could then be compared with the theoretical energy changes and polarisations.

# Chapter 3

## Predicted results

### 3.1 Wavelengths of the central spectral lines

By using an energy level diagram for mercury [5], a list of known wavelengths for the spectral lines of mercury [7], a list of known states for mercury [4] and by checking the level of splitting for each it was possible to ascertain that the wavelengths of the central spectral lines (i.e. the line obtained when there is no magnetic field acting on the sample), and the corresponding energy level shift in each case, were as given in table 3.1.

Colour	Wavelength	Transition	
		from:	to:
Violet	$404.656 \times 10^{-9}$ m	$^3S_1$	$^3P_0$
Green	$546.074 \times 10^{-9}$ m	$^3S_1$	$^3P_2$
Blue	$435.833 \times 10^{-9}$ m	$^3S_1$	$^3P_1$
Yellow	$579.066 \times 10^{-9}$ m	$^1D_1$	$^1P_1$

Table 3.1: Wavelengths of the central line for each colour.

### 3.2 Energy level diagrams

Knowing which states the transitions were between it was then possible to draw an energy diagram of the nature of the splitting and the allowed transitions (see table 1.1) shown. Figures 3.1a and 3.1b show some samples of the energy level diagrams for violet and green respectively.

### 3.3 Energy changes

Using equation 1.2, it was possible to work out the expected energy changes between the peaks. These are shown in table 4.1. The value for the magnetic field was measured by taking multiple readings between the coils using a handheld gaussmeter and was found to be:

$$B = 1.2(\pm 0.1) \text{ T} \tag{3.1}$$

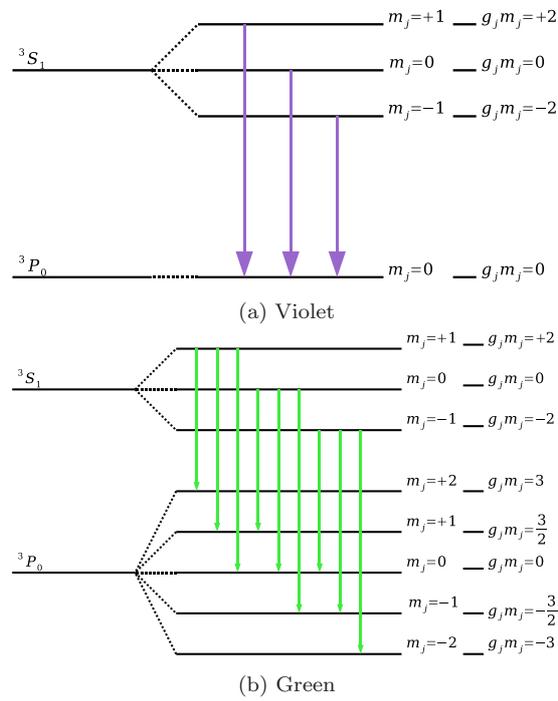


Figure 3.1: Energy level diagrams.

The errors on values were found by applying the general error propagation equation.[8]

$$\Delta f = \frac{df}{dx_1} \Delta x_1 + \frac{df}{dx_2} \Delta x_2 + \dots + \frac{df}{dx_n} \Delta x_n \quad (3.2)$$

### 3.4 Polarisation and Intensity

Using the information in table 1.2, it can be seen that the polarisation of each set of lines should be as shown in figures 3.2a - 3.2d.

Lines from a certain transition will be more, or less, intense than lines from another transition depending on the change that takes place. Table 3.2 gives the rules for these intensity changes.

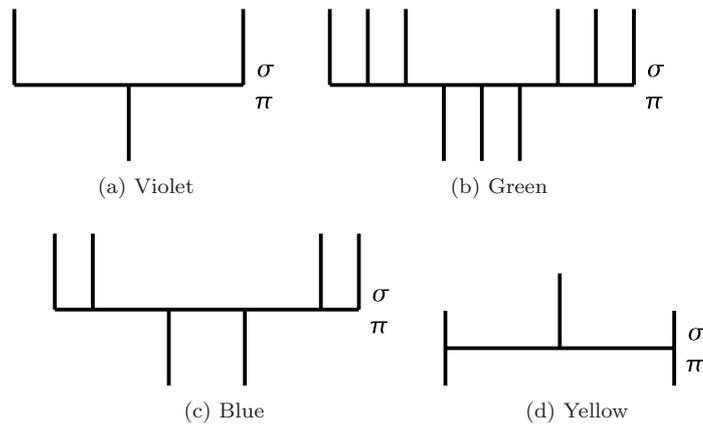


Figure 3.2: Expected polarisations. Notice that there are 2 contributions to the outside lines for yellow, one being a  $\pi$ -line and the other a  $\sigma$ -line. The  $\pi$  contribution however is the more intense contribution (this is not indicated in the diagram), and the  $\sigma$  contribution should be zero due to the intensity rules shown in table 3.2.

$j \rightarrow j$	$m_j \rightarrow m_j$	$I = 4am_j^2$
	$m_j \rightarrow m_j \pm 1$	$I = a(j \pm m_j + 1)(j \mp m_j)$
$j \rightarrow j \pm 1$	$m_j \rightarrow m_j$	$I = 4b(j + m_j + 1)(j - m_j + 1)$
	$m_j \rightarrow m_j \pm 1$	$I = b(j \pm m_j + 1)(j \pm m_j + 2)$

Table 3.2: Intensity rules for transitions.

# Chapter 4

## Results

Figures 4.1 and 4.2 show samples of the images obtained and the graphs of the intensity for each pixel through a slice of the image.

### 4.1 Energy changes

Using equations 2.1 and 2.2 it was possible to find the wavelength and energy changes between the central line and the other lines. The errors on the values recorded were found using equation 3.2.

Table 4.1 shows the recorded change in energy between the centre line and each other line for all four colours.

### 4.2 Polarisation

Figure 4.3 show some samples of the images acquired when using the polarising filter.

All of the images obtained for the polarisation followed the expected results in figure 3.2 with the exception of the yellow lines, which seemed to be polarised in the exact opposite way.

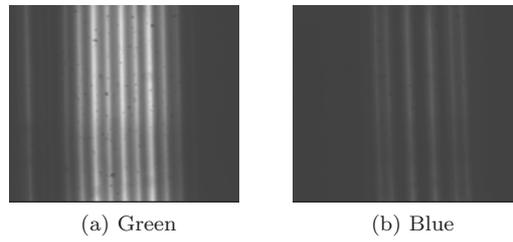


Figure 4.1: Samples of the images obtained.

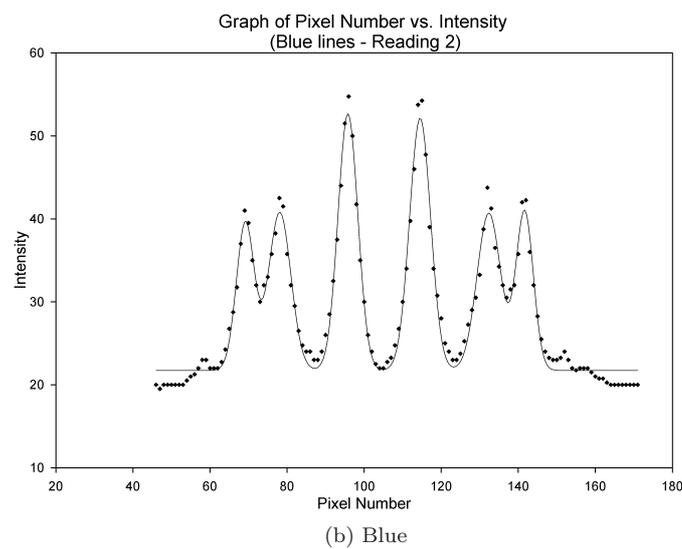
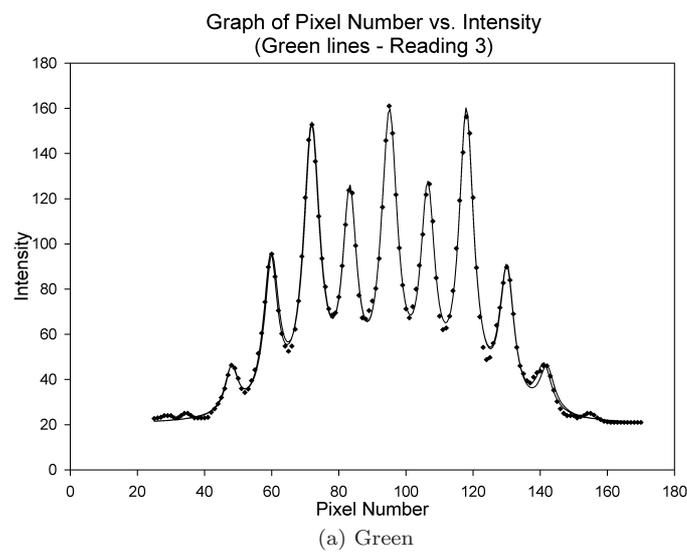


Figure 4.2: Samples of the graphs of intensity vs. pixel number for a slice through the images in figure 4.1.

Line:	$\Delta E_{\text{measured}} / \mu_B B$ J	$\Delta E_{\text{expected}} / \mu_B B$ J
1	$-2.8 \pm 0.2$	-2
2	$0 \pm 0$	0
3	$+2.7 \pm 0.2$	2

(a) Violet

Line:	$\Delta E_{\text{measured}} / \mu_B B$ J	$\Delta E_{\text{expected}} / \mu_B B$ J
1	$-2.8 \pm 0.2$	-2
2	$-2.1 \pm 0.2$	$-\frac{3}{2}$
3	$-1.4 \pm 0.1$	-1
4	$-0.71 \pm 0.06$	$-\frac{1}{2}$
5	$0 \pm 0$	0
6	$+0.69 \pm 0.06$	+2
7	$+1.4 \pm 0.1$	$+\frac{3}{2}$
8	$+2.1 \pm 0.2$	+1
9	$+2.8 \pm 0.2$	$+\frac{1}{2}$

(b) Green

Line:	$\Delta E_{\text{measured}} / \mu_B B$ J	$\Delta E_{\text{expected}} / \mu_B B$ J
1	$-2.7 \pm 0.2$	-2
2	$-2.1 \pm 0.2$	$-\frac{3}{2}$
3	$-0.71 \pm 0.06$	$-\frac{1}{2}$
4	$+0.71 \pm 0.06$	$+\frac{1}{2}$
5	$+2.1 \pm 0.2$	$+\frac{3}{2}$
6	$+2.7 \pm 0.2$	+2

(c) Blue

Line:	$\Delta E_{\text{measured}} / \mu_B B$ J	$\Delta E_{\text{expected}} / \mu_B B$ J
1	$-1.6 \pm 0.1$	-1
2	$0 \pm 0$	0
3	$+1.6 \pm 0.1$	+1

(d) Yellow

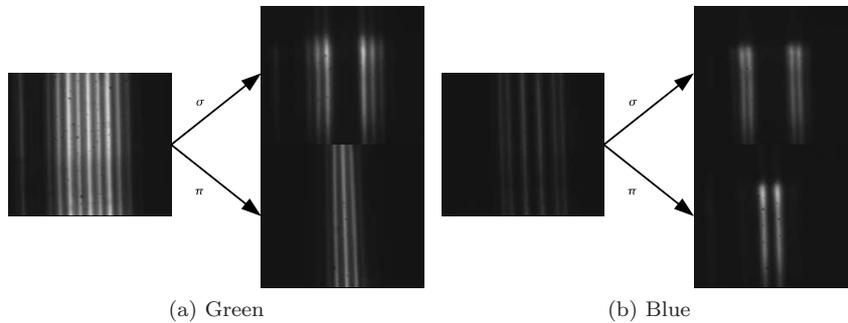
Table 4.1: The recorded and predicted changes in energy between the central line and the other lines in units of  $\mu_B B$  J.

Figure 4.3: Sample images showing the effect of polarisation on the lines.

# Chapter 5

## Conclusion

### 5.1 Energy changes

All of the measured wavelength changes are to the same order of magnitude of the predicted values however none of them fall within the errors obtained. It appears that the further away from the central peak the line of interest is, the further away from the predicted value it is.

There are a number of possible reasons for this being the case and some of these will be discussed here.

#### 5.1.1 Possible error caused by the use of *SigmaPlot*

The values recorded by *SigmaPlot* may have been incorrect, this would have introduced the apparently systematic error seen.

*SigmaPlot* caused a number of problems during the data analysis phase of this experiment and as such it would be advised to use a different package for graphing and analysis of data in further experiments. Possible software packages include *Matlab* and *fityk*.

#### 5.1.2 Possible error caused by the value for ascertaining the frequency shift

The value quoted in the script for ascertaining the frequency shift from the difference in pixels may have been incorrect. This would seem to be the probable cause of this error. It can be seen that if this was wrong every time you added the wavelength difference between two lines to find the distance of a line from the centre line you would add this error repeatedly, thus causing the increasing error seen.

This value could be affected by movement of the Echelle grating, or the other components of the setup, and would change if the Echelle grating didn't have the same specifications as the one for which this value was calculated. Any change in these criteria could easily have occurred.[6]

This could be rectified by recalculating this value.

### 5.1.3 Possible error from optical aberrations

Any aberrations on the optics used could have introduced an error in the results of the type seen.

This could be rectified by either cleaning and/or regrinding the optics, or using a completely different set of optics.

### 5.1.4 Possible error in the theory

The transitions occurring may not have been strictly  $LS$ -coupling related, there could have been  $jj$ -coupling involved. This is known to be the case at least for the yellow lines.[5]

Rectifying this would require a recalculation of the theoretical energy change (and polarisation) values to include the effect of  $jj$ -coupling.

## 5.2 Polarisation

With the exception of the yellow lines, the polarisations all follow the expected cases seen in figure 3.2.

There are a number of reasons that this could be the case that this could be the case and some will be discussed here.

### 5.2.1 Possible human error

The most obvious, and most easily rectifiable, possibility is that the polarising filter was mistakenly inserted at the wrong angle.

Simply retaking the measurement will correct this error.

### 5.2.2 Possible error caused by the effect of $jj$ -coupling

The known  $jj$ -coupling influence on the yellow lines[5] may affect the polarisation of the emitted light.

This could be rectified by reappraisal of the theory, taking into account the  $jj$ -coupling influence, and further investigation.

# Bibliography

- [1] A. I. M. Rae. *Quantum Mechanics*, chapter 6.5. Taylor & Francis, 4<sup>th</sup> edition, 2002.
- [2] B. H. Bransdon and C. J. Joachain. *Quantum Mechanics*, chapter 12.2. Pearson Prentice Hall, 2<sup>nd</sup> edition, 2000.
- [3] B. H. Bransdon and C. J. Joachain. *Physics of Atoms and Molecules*, chapter 9.9. Prentice Hall, 2<sup>nd</sup> edition, 2003.
- [4] Middlebury College. The Zeeman Effect in Atomic Mercury. Website, 1992-1993. <http://cat.middlebury.edu/~PHManual/zeeman.html>.
- [5] J. Stoltenberg, D. Pengra, and R. Van Dyck. Zeeman Effect, February 2006.
- [6] N. T. Skipper *et al.* High Resolution Spectrograph (the Zeeman Effect). UCL Department of Physics and Astronomy, September 2007.
- [7] D. R. Lide, editor. *CRC Handbook of Chemistry and Physics*. CRC-Press, 79<sup>th</sup> edition, August 1998.
- [8] Louis Lyons. *A Practical Guide to Data Analysis for Physical Science Students*. Cambridge, 1991.